

Discrete mathematics sussana solution manual Academic PDF

Discrete Mathematics Sussana Solution Manual: Your Ultimate Guide to Mastering Discrete Mathematics

If you're venturing into the world of discrete mathematics, chances are you've encountered the *Discrete Mathematics Sussana Solution Manual*? a vital resource for students and professionals seeking clarity and confidence in this complex subject. Discrete mathematics forms the foundation for many fields, including computer science, cryptography, and information theory. Having access to a comprehensive solution manual like Sussana's can significantly enhance your learning experience by providing detailed explanations, step-by-step solutions, and reinforcing key concepts. In this article, we'll explore the importance of the **Discrete Mathematics Sussana Solution Manual**, how it can aid your studies, and practical tips for maximizing its benefits.

Understanding the Significance of the Discrete Mathematics Sussana Solution Manual

Discrete mathematics involves a variety of topics such as logic, set theory, combinatorics, graph theory, and algorithms. These topics often present challenges due to their abstract nature and problem-solving requirements. The *Solution Manual* by Sussana serves as an invaluable tool for students to bridge the gap between theory and application.

Why Use a Solution Manual?

- **Clarification of Concepts:** Complex topics are broken down into understandable steps.
- **Practice and Reinforcement:** Working through solutions helps solidify knowledge.
- **Time Efficiency:** Quickly verify answers and learn alternative problem-solving methods.
- **Preparation for Exams:** Practice with solutions prepares students for assessments and reduces exam anxiety.

Features of the Sussana Solution Manual

- **Detailed Step-by-Step Solutions:** Each problem is addressed with comprehensive explanations.
- **Coverage of Core Topics:** Includes logic, proofs, combinatorics, graph theory, and more.

- **Alignment with Course Content:** Designed to complement the Sussana textbook and coursework.
- **Accessible Format:** Available in print and digital formats for convenient access.

How to Effectively Use the Discrete Mathematics Sussana Solution Manual

Maximizing the benefits of the solution manual requires strategic use. Here are some practical tips:

Active Problem Solving

- Attempt problems on your own first. Use the manual to check your work or to understand the solution if you're stuck.
- Analyze step-by-step solutions to grasp the underlying logic and methodology.
- Practice variations of problems to deepen your understanding.

Integrate with Study Routine

- Use the manual as a supplementary resource alongside your textbook.
- Review solutions after completing assignments to identify areas needing improvement.
- Incorporate solution review sessions into your study schedule for consistent learning.

Enhance Critical Thinking Skills

- Compare your solutions with those provided to identify alternative approaches.
- Try to derive solutions independently before consulting the manual.
- Use the manual to understand common pitfalls and misconceptions.

Key Topics Covered in the Discrete Mathematics Sussana Solution Manual

A comprehensive solution manual like Sussana's addresses a wide spectrum of topics essential to mastering discrete mathematics. Here's an overview:

Logic and Boolean Algebra

- Propositional logic

- Logical equivalences
- Predicates and quantifiers
- Boolean functions and simplification techniques

Set Theory and Relations

- Sets, subsets, and set operations
- Cartesian products
- Relations and their properties
- Equivalence relations and partitions

Combinatorics

- Counting principles
- Pigeonhole principle
- Permutations and combinations
- Recursion and recurrence relations

Graph Theory

- Graphs and their representations
- Traversals: DFS and BFS
- Tree structures
- Graph coloring and matching

Algorithms and Complexity

- Basic algorithm design techniques
- Recursion and iterative solutions
- Analysis of algorithm efficiency

Where to Find the Discrete Mathematics Sussana Solution Manual

Accessing the **Discrete Mathematics Sussana Solution Manual** is straightforward through various channels:

Official Publishers and Retailers

- Check the official publisher's website for authorized copies.
- Major online retailers such as Amazon, Barnes & Noble, or specialized academic bookstores.

Educational Platforms and Libraries

- University libraries often provide physical or digital copies for students.
- Educational platforms like Chegg or Course Hero may host solutions manuals?ensure compliance with academic integrity policies.

Note on Ethical Use

While solution manuals are excellent learning aids, they should complement your independent study rather than replace it. Always strive to understand the reasoning behind each solution to truly master discrete mathematics.

Conclusion: Why the Discrete Mathematics Sussana Solution Manual is Indispensable

In the journey of mastering discrete mathematics, having a reliable resource like the *Sussana Solution Manual* can make a significant difference. It provides clarity, reinforces learning, and builds confidence in solving complex problems. Whether you're a student preparing for exams, a professional brushing up on concepts, or an educator seeking supplementary materials, this solution manual is an invaluable asset.

Remember, the key to success lies in active learning?try problems on your own first, use the manual to verify and understand solutions, and always aim to grasp the underlying principles. Coupled with consistent practice and curiosity, the **Discrete Mathematics Sussana Solution Manual** will support your path toward mastery in one of the most fundamental areas of mathematics and computer science.

Start leveraging this resource today and unlock your potential in discrete mathematics!

Discrete Mathematics Susana Solution Manual: A Comprehensive Guide for Students and Educators

In the realm of computer science and mathematics, discrete mathematics sussana solution manual serves as an invaluable resource for students seeking to deepen their understanding of complex concepts and for educators aiming to facilitate effective teaching. This solution manual, often associated with Susana's discrete mathematics textbook, provides detailed step-by-step solutions to a wide array of problems, ensuring learners can verify their work and grasp the underlying principles

that govern topics such as logic, set theory, combinatorics, graph theory, and algorithms.

Understanding the Significance of the Discrete Mathematics Susana Solution Manual

Discrete mathematics forms the backbone of theoretical computer science, underpinning algorithms, data structures, cryptography, and more. The discrete mathematics sussana solution manual is designed to complement the textbook by offering solutions that clarify complex problems, reinforce learning, and build confidence in tackling challenging exercises.

Key benefits include:

- Clarification of complex concepts: Step-by-step solutions break down intricate problems into manageable parts.
- Self-assessment: Students can compare their solutions and identify areas needing improvement.
- Time-saving resource: Quick access to solutions accelerates study sessions and exam preparations.
- Teaching aid: Educators can use it to prepare lectures and design assignments effectively.

Navigating the Contents of the Susana Solution Manual

The solution manual typically covers chapters aligned with the textbook, which might include:

- Logic and propositional calculus
- Predicates and quantifiers
- Set theory and relations
- Functions and algorithms
- Number theory
- Combinatorics
- Graph theory
- Boolean algebra

Each section provides detailed solutions with explanations, diagrams, and often, alternative methods for solving problems.

Effective Strategies for Using the Solution Manual

To maximize its benefits, students should approach the solution manual strategically:

- Attempt problems independently first: Use the manual as a learning aid rather than a shortcut.
- Compare solutions: Study the step-by-step reasoning to understand problem-solving techniques.
- Analyze different methods: If multiple solutions exist, explore all to deepen understanding.
- Use as a study guide: Review solutions after attempting exercises to reinforce concepts.
- Seek clarification: If a solution is unclear, consult additional resources or instructors.

Key Topics and Sample Problem Solutions

- Logic and Propositional Calculus

Sample Problem: Prove that the statement $\neg (p \wedge q)$ implies $\neg p$.

Solution Outline:

- Use a truth table or logical equivalences.
- Demonstrate that whenever $\neg (p \wedge q)$ is true, $\neg p$ must be true.
- Conclude that the implication holds universally.

Tips: Understanding logical equivalences such as the implication and conjunction rules is crucial here.

- Set Theory and Relations

Sample Problem: Show that the union of two equivalence relations is not necessarily an equivalence relation.

Solution Outline:

- Recall the properties of equivalence relations: reflexivity, symmetry, transitivity.
- Provide counterexamples where union loses transitivity.
- Conclude that union of equivalence relations may fail to be an equivalence relation.

Tips: Visualize relations with diagrams to better understand properties.

- Functions and Algorithms

Sample Problem: Find the inverse of a bijective function $f: \mathbb{R} \rightarrow \mathbb{R}$, defined by $f(x) = 3x + 5$.

Solution Outline:

- Set $y = 3x + 5$.
- Solve for x : $x = \frac{y - 5}{3}$.
- Write the inverse function as $f^{-1}(y) = \frac{y - 5}{3}$.

Tips: Practice with various functions to develop intuition for inverses.

Tips for Mastering Discrete Mathematics with the Susana Solution Manual

- Practice Regularly: Consistent problem-solving enhances retention.
- Understand, Don't Memorize: Focus on grasping concepts rather than rote memorization.
- Work in Groups: Collaborate with peers for diverse perspectives.
- Utilize Additional Resources: Refer to online tutorials, videos, and forums.
- Seek Clarification: Don't hesitate to ask instructors about solutions that are unclear.

Ethical Use of the Solution Manual

While the discrete mathematics sussana solution manual is a powerful tool, ethical use is critical:

- Use it as a learning aid, not a shortcut for assignments.
- Attempt problems independently before consulting solutions.
- Give credit to original authors and resources when discussing solutions.

Responsible use ensures genuine learning and academic integrity.

Final Thoughts

The discrete mathematics sussana solution manual is more than just a collection of answers; it is a comprehensive guide that fosters critical thinking, problem-solving skills, and a deeper appreciation of discrete structures. When used thoughtfully, it can significantly enhance your mastery of the subject, preparing you for advanced studies and professional applications in computer science, mathematics, and engineering.

Whether you are a student striving to excel or an educator seeking effective teaching tools,

leveraging this solution manual can make your journey through discrete mathematics more manageable, insightful, and rewarding. Remember, the goal is not just to find the right answers but to understand the logic and reasoning that lead there. Happy studying!

QuestionAnswer What is the main focus of the 'Discrete Mathematics Susanna Solution Manual'? The manual provides detailed solutions to problems from Susanna's Discrete Mathematics textbook, covering topics like logic, set theory, combinatorics, graph theory, and algorithms to aid students in understanding and mastering the subject. How can the 'Discrete Mathematics Susanna Solution Manual' help students improve their problem-solving skills? By offering step-by-step solutions and explanations for various exercises, the manual helps students understand the underlying concepts, develop analytical skills, and apply methods effectively to similar problems. Is the 'Discrete Mathematics Susanna Solution Manual' suitable for self-study? Yes, it is designed to complement the textbook and is a valuable resource for self-learners, providing clear solutions and guidance to reinforce understanding outside classroom settings. Where can I find the 'Discrete Mathematics Susanna Solution Manual' legally? You can access the manual through authorized educational platforms, university libraries, or by purchasing official copies from publishers or bookstores that offer legitimate educational resources. Are there any online forums or communities where students discuss the 'Discrete Mathematics Susanna Solution Manual'? Yes, platforms like Reddit, Stack Exchange, and dedicated study groups on social media often discuss solutions and share insights related to Susanna's Discrete Mathematics textbook and its manual. Does the 'Discrete Mathematics Susanna Solution Manual' cover recent updates or editions? The manual typically aligns with the latest editions of Susanna's textbook, but it's important to check the edition compatibility to ensure accurate solutions. Can the 'Discrete Mathematics Susanna Solution Manual' be used for exam preparation? Absolutely, as it provides detailed solutions that help students understand concepts deeply, making it a useful resource for reviewing and practicing for exams. Are there digital versions of the 'Discrete Mathematics Susanna Solution Manual' available? Yes, digital copies may be available through authorized e-book platforms or educational resource websites, offering convenient access for students. What should students do if they find discrepancies in the 'Discrete Mathematics Susanna Solution Manual'? Students are advised to consult their instructors, cross-reference with the textbook, or seek help from online academic communities to clarify and resolve any inconsistencies.

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Discrete probability models and methods probabili

discrete probability models and methods probabili form a foundational pillar in the field of probability theory and statistics. These models are essential for understanding and analyzing situations where the set of possible outcomes is countable or finite, such as rolling dice, flipping coins, or drawing cards from a deck. By employing discrete probability models, statisticians and mathematicians can quantify uncertainty, make predictions, and derive meaningful inferences from data. This article delves into the core concepts, common models, methods for analysis, and practical applications of discrete probability models and methods probabili.

Understanding Discrete Probability Models

Discrete probability models are mathematical frameworks used to describe random experiments with discrete outcomes. These models assign probabilities to each possible outcome, adhering to specific axioms that ensure the probabilities make sense within the context of real-world scenarios.

Basic Concepts and Definitions

- **Sample Space (Ω):** The set of all possible outcomes of a random experiment. For discrete models, Ω is countable (finite or countably infinite).
- **Event:** Any subset of the sample space. Events can be simple (single outcome) or compound (multiple outcomes).
- **Probability Measure (P):** A function that assigns a probability to each event, satisfying the axioms:
 - Non-negativity: $P(E) \geq 0$ for any event E .
 - Normalization: $P(\Omega) = 1$.
 - Additivity: For mutually exclusive events $E_1, E_2, \dots$, $P(E_1 \cup E_2 \cup \dots) = P(E_1) + P(E_2) + \dots$

Probability Distributions in Discrete Models

Discrete probability distributions specify how the total probability is distributed among the outcomes. Some of the most common discrete distributions include:

- **Uniform Distribution:** All outcomes are equally likely. For a finite set of n outcomes, $P(X = x) = 1/n$.
- **Bernoulli Distribution:** Describes a single trial with two outcomes: success (with probability p) and failure (with probability $1-p$).
- **Binomial Distribution:** Models the number of successes in n independent Bernoulli trials.
- **Poisson Distribution:** Describes the number of events occurring in a fixed interval of time or

space when events occur independently.

Common Discrete Probability Models

Each model has specific contexts where it best applies and unique properties that facilitate analysis and computation.

Uniform Distribution

The simplest form, where each outcome has equal probability. Used when outcomes are equally likely, such as rolling a fair die or selecting a random card from a deck.

Bernoulli Distribution

Representing the simplest case of a binary experiment, the Bernoulli distribution models outcomes like coin flips or yes/no responses.

- Probability mass function (PMF): $P(X = x) = p^x (1-p)^{(1-x)}$, where $x \in \{0, 1\}$.

Binomial Distribution

Extends Bernoulli trials to n independent experiments, each with success probability p . It models the total number of successes.

- PMF: $P(X = k) = C(n, k) p^k (1-p)^{(n-k)}$, for $k = 0, 1, \dots, n$.

Poisson Distribution

Ideal for modeling rare events over a continuous domain, such as the number of emails received in an hour or decay events in radioactive substances.

- PMF: $P(X = k) = (\lambda^k e^{-\lambda}) / k!$, for $k = 0, 1, 2, \dots$

Methods for Analyzing Discrete Probability Models

Analyzing discrete models involves calculating probabilities, expected values, variances, and other statistical measures. Several methods and tools are commonly used.

Probability Calculations

Calculations often rely on the probability mass function (PMF). For compound events, the sum or convolution of individual probabilities is used.

Expected Value and Variance

These measures provide insights into the average outcome and variability:

- **Expected Value (Mean):** $E[X] = \sum x \cdot P(X = x)$.
- **Variance:** $\text{Var}(X) = E[(X - E[X])^2] = \sum (x - E[X])^2 P(X = x)$.

Conditional Probability

Understanding the probability of an event given another event is crucial, especially in complex models.

- $P(A | B) = P(A \cap B) / P(B)$, provided $P(B) > 0$.

Independence

Two events A and B are independent if $P(A \cap B) = P(A) P(B)$. Independence simplifies calculations and is a key assumption in many models.

Using Generating Functions

Probability generating functions (PGFs) and moment generating functions (MGFs) are powerful tools for deriving moments and distributions of sums of discrete random variables.

Practical Applications of Discrete Probability Models

These models are instrumental across various industries and fields, enabling better decision-making and risk assessment.

Gaming and Gambling

- Understanding the probabilities in card games, dice, roulette, and lotteries.
- Calculating odds and expected winnings.

Quality Control and Manufacturing

- Modeling defect rates in production lines.
- Determining the likelihood of a certain number of defective items.

Communication Systems

- Analyzing packet loss, error rates, and the number of failures in network systems.

Biology and Epidemiology

- Modeling the number of mutations, disease outbreaks, or survival counts.

Business and Economics

- Forecasting demand, customer arrivals, and inventory levels.

Advanced Topics in Discrete Probability Methods

Beyond basic models, advanced methods enhance analysis and modeling capabilities.

Markov Chains

- Modeling systems that transition between states with certain probabilities.
- Applications include weather prediction, stock market analysis, and queuing systems.

Poisson Processes

- Modeling the occurrence of events over continuous time or space.
- Useful in telecommunications, traffic flow, and reliability engineering.

Multivariate Discrete Distributions

- Handling multiple correlated discrete variables.

- Examples include multinomial distributions and joint probability tables.

Conclusion

Discrete probability models and methods probabilistic are vital tools for quantifying uncertainty in situations with countable outcomes. They provide a structured approach to calculating probabilities, understanding distributions, and deriving statistical measures essential for decision-making across diverse fields. Mastery of these models enables analysts and researchers to interpret data accurately, develop predictive models, and implement effective strategies in contexts ranging from gaming and manufacturing to healthcare and finance. As the landscape of data analysis continues to evolve, discrete probability models remain a cornerstone of statistical reasoning and probabilistic modeling.

Discrete Probability Models and Methods Probabilistic: An In-Depth Review

In the realm of probability theory, discrete probability models serve as foundational tools for understanding and analyzing phenomena characterized by countable outcomes. From the simple toss of a coin to complex network models, discrete probability models underpin a vast array of applications across sciences, engineering, economics, and social sciences. This review provides a comprehensive exploration of discrete probability models and methods probabilistic, tracing their theoretical underpinnings, key methodologies, practical applications, and recent advancements.

Introduction to Discrete Probability Models

Discrete probability models describe random experiments with a finite or countably infinite set of possible outcomes. Unlike continuous models, which deal with outcomes in an uncountable space, discrete models assign probabilities to individual outcomes, often facilitating exact calculations and interpretations.

Fundamental Concepts

- Sample Space (Ω): The set of all possible outcomes. For discrete models, Ω is countable.
- Event: A subset of the sample space, representing a collection of outcomes.
- Probability Measure (P): A function assigning probabilities to events, satisfying axioms of non-negativity, normalization, and countable additivity.
- Probability Mass Function (PMF): For each outcome ω in Ω , $P(\omega)$ gives its probability; the sum over all outcomes equals 1.

Common Discrete Probability Distributions

- Bernoulli Distribution: Models a single trial with two outcomes, success with probability p , failure with $1-p$.
- Binomial Distribution: Number of successes in n independent Bernoulli trials.
- Geometric Distribution: Number of trials until the first success.
- Negative Binomial Distribution: Number of trials until a fixed number of successes.
- Poisson Distribution: Count of events occurring in a fixed interval, often modeling rare events.

Methodologies and Probabilistic Techniques

Understanding and applying discrete probability models involve various methods that enable analysts to derive probabilities, expectations, variances, and other statistical measures.

1. Enumeration and Combinatorics

Many discrete models rely on combinatorial principles to count the number of favorable outcomes. Techniques include:

- Counting arrangements (permutations)
- Counting subsets (combinations)
- Applying the inclusion-exclusion principle

These methods are essential in deriving explicit formulas for PMFs.

2. Generating Functions

Generating functions encode probability distributions into algebraic forms, facilitating analysis and derivation of moments:

- Probability Generating Function (PGF): $G_X(s) = E[s^X]$
- Use Cases: Deriving moments, convolutions, and limit distributions.

3. Conditional Probability and Bayes' Theorem

Conditional probability is central to models involving dependencies or updates based on new information. Bayesian methods extend discrete models by updating probabilities as evidence accumulates.

4. Markov Chains and Stochastic Processes

Discrete models often extend to sequences of random variables with the Markov property, where the future state depends only on the current state. Applications include:

- Modeling queues
- Population dynamics
- Random walks

Advanced Topics and Methods Probabili in Discrete Settings

As the field has evolved, new techniques and models have expanded the scope of discrete probability analysis.

1. Discrete Empirical Distributions

- Used when the underlying distribution is unknown.
- Empirical probabilities are estimated directly from data.
- Essential in non-parametric inference.

2. Approximation Methods

- Poisson Approximation: Approximates binomial distributions when p is small, and n is large.
- Normal Approximation: For large n , binomial and other discrete distributions approximate the normal distribution, via the Central Limit Theorem.

3. Monte Carlo and Simulation Methods

- Use computational algorithms to simulate discrete random variables.
- Useful when analytical solutions are complex or intractable.
- Enable estimation of probabilities, expectations, and variances.

Applications of Discrete Probability Models

Discrete probability models are ubiquitous across disciplines, with applications including:

- Quality Control: Modeling defect counts in manufacturing.
- Epidemiology: Counting disease cases or transmission events.

- Information Theory: Modeling message counts, packet arrivals.
- Economics: Modeling discrete choices, market states.
- Computer Science: Algorithm analysis, randomized algorithms, network modeling.

Recent Advancements and Research Directions

The increasing complexity of real-world data has spurred several recent developments:

1. Discrete Bayesian Models

Bayesian approaches allow for flexible incorporation of prior knowledge and updating beliefs with new data. Discrete Bayesian models are increasingly applied in areas like natural language processing and bioinformatics.

2. High-Dimensional Discrete Data

Handling high-dimensional discrete data (e.g., categorical data with many levels) requires specialized modeling techniques, such as hierarchical models and graphical models.

3. Discrete Survival and Event Models

Extensions of survival analysis to discrete time frames facilitate modeling events like system failures or disease onset at specific intervals.

4. Machine Learning and Discrete Models

Algorithms such as decision trees, discrete latent variable models, and probabilistic graphical models leverage discrete probability methods for classification, clustering, and inference tasks.

Challenges and Future Perspectives

While discrete probability models are powerful, several challenges persist:

- Scalability: As data size and complexity grow, computational efficiency becomes critical.
- Model Selection: Choosing appropriate distributions and parameters requires robust criteria and methods.
- Interpretability: Ensuring models remain understandable for practical decision-making.
- Integration with Continuous Models: Hybrid models that combine discrete and continuous variables are increasingly relevant.

Future research is poised to focus on developing more efficient algorithms, richer models for complex data, and integrating discrete probability methods with other statistical and machine learning frameworks.

Conclusion

Discrete probability models and methods form a vital part of the statistical toolkit for analyzing countable random phenomena. Their theoretical elegance, combined with practical versatility, makes them indispensable across scientific disciplines. As data complexity continues to escalate, ongoing innovations in modeling techniques, computational algorithms, and applications will ensure their relevance and utility well into the future. Embracing these models' theoretical foundations and methodological advancements will remain essential for researchers and practitioners aiming to extract meaningful insights from discrete data.

QuestionAnswer What are discrete probability models and how are they used in probability theory? Discrete probability models are mathematical frameworks that describe the likelihood of different outcomes in scenarios where the possible outcomes are countable or finite. They are used to calculate probabilities, analyze distributions, and predict outcomes in applications like games, queues, or sampling processes. What is the significance of probability mass functions (PMFs) in discrete models? Probability mass functions (PMFs) specify the probability of each possible outcome in a discrete random variable. They are fundamental in discrete probability models because they allow us to compute probabilities, expected values, and variances for discrete distributions. How do the Binomial and Poisson distributions serve as core discrete probability models? The Binomial distribution models the number of successes in a fixed number of independent Bernoulli trials with the same probability, while the Poisson distribution models the number of events occurring in a fixed interval of time or space when events happen independently at a constant average rate. Both are essential for modeling count data in various fields. What methods are commonly used to estimate parameters in discrete probability models? Methods such as Maximum Likelihood Estimation (MLE), Method of Moments, and Bayesian inference are commonly used to estimate parameters in discrete probability models, providing the best-fit values based on observed data. How can discrete probability models be applied in real-world scenarios? Discrete models are used in areas like quality control (defect counts), insurance (claim counts), queuing systems (number of customers), and genetics (number of genetic traits), helping to analyze, predict, and make decisions based on count-based data. What are the key assumptions underlying discrete probability models? Key assumptions typically include independence of events, fixed probabilities or rates, and the countable nature of outcomes. These assumptions ensure the models accurately reflect the stochastic processes they represent. How do discrete probability methods differ from continuous probability methods? Discrete probability methods deal with countable outcomes and use PMFs to assign probabilities, whereas continuous methods handle uncountably infinite outcomes using probability density functions (PDFs). The mathematical tools and interpretations differ accordingly. What are some challenges in working with discrete probability

models and how can they be addressed? Challenges include small sample sizes, model misspecification, and dependencies between events. These can be addressed by collecting sufficient data, validating models through goodness-of-fit tests, and incorporating dependence structures or more complex models like Markov chains.

Related keywords: discrete probability, probability distributions, combinatorics, random variables, binomial distribution, geometric distribution, probability mass function, joint probability, Markov chains, probability theory

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